

The Unification of Repetitive Measurements in Metrology and Quantum Mechanics  
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Abstract: Repetitive single measurements in metrology practice appear equal. Repetitive single measurements in quantum mechanics (QM), both practice and theory, are not equal. Beginning before 1935, this and other measurement discrepancies have been studied extensively from a QM perspective without a satisfactory resolution. The current unappealing opinion is that metrology and QM are not correlated. Applying a metrology perspective based upon set theory, this paper develops finite (describing metrology and QM practice) and transfinite (describing QM theory) examples of Relative Measurement Theory (RMT). RMT changes Representational Measurement Theory (the current theory of measurements), identifies that all repetitive single measurement results in practice are not equal at high precision, resolves the current discrepancies in both metrology and QM, and correlates metrology with QM.

## 1.0 Introduction

A list of current discrepancies in metrology and quantum mechanics (QM):

- How does Gödel's incompleteness theorems apply to measurement systems?<sup>1</sup>
- How do measurement results meet Tarski's undefinability requirement?<sup>2</sup>
- How does one observable have both wave and particle properties?<sup>3</sup>
- Why do normal distributions appear in repetitive measurements?<sup>4</sup>
- What in metrology is equivalent to QM uncertainty?<sup>5</sup>
- A QM observable in one state appears as a distribution of states.<sup>6</sup>
- The QM description of physical reality is not complete.<sup>7</sup>

There have been many QM proposals addressing the last three discrepancies, but these proposals have not been widely accepted.<sup>8</sup> All seven discrepancies and other measurement issues are resolved (see Section 10) by Relative Measurement Theory (RMT).<sup>9</sup> RMT defines a physical measurement result relative to a coordinate system of exact System International (SI) units of meters, kilograms, seconds, etc. (i.e., the independent reference) which define physical properties.<sup>10</sup> That is, RMT follows existing metrology practice, which is not the same as the current QM perception of metrology, often termed: "classical measurements."

This paper is organized as follows:

- 1.0 Introduction
- 2.0 Metrology (calibration to independent references)
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- 8.0 RMT correlates with QM theory
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- 10. Resolving the remaining issues
- 11. Conclusions.

## 2.0 Metrology (calibration to independent references)

In 1822, L. Euler wrote, "It is not possible to determine or measure one quantity other than by assuming that another quantity of the same type is known and determining the ratio between the quantity being measured and that quantity."<sup>11</sup> Euler states that all measurement results, theory and practice, are relative to a reference.

In 1891, J. C. Maxwell<sup>12</sup> defined one relative measurement result as a quantity (1).

$$\text{quantity} = \text{numerical value } (x) \text{ times unit } (u) \quad (1)$$

Maxwell's numerical value ( $x$ ) is relative to the units ( $u$ ) of a measurement instrument.<sup>i</sup> That is, all  $u$  are assumed to be equal to  $U$  a unit standard (i.e., the reference) without calibration. QM follows (1) treating calibration as only empirical. Metrology<sup>13</sup> requires that the units of a measurement instrument be calibrated to a unit standard in a coordinate system, i.e.,  $u = U$  after calibration, treating calibration as required in metrology practice. This difference in the requirement for calibration is the beginnings of the discrepancies between metrology and QM measurements.

Calibration includes three processes: reference, correlation, and correction, all of which are shown to have theoretical impact:

- Calibration establishes a rigorous (not assumed as Euler indicated) reference ( $U$ ) defining the property of a quantity.
- Calibration determines the precision of the numerical value of the measurement instrument units relative to  $U$ .
- Calibration corrects for the measurement instrument's unit precision relative to  $U$ .

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<sup>i</sup> In this paper a *measurement instrument* is empirical and has a scale of ordinal units not defined as equal. A *measurement apparatus* may be empirical (ordinal units not defined as equal) or theoretical (equal ordinals).

Metrology practice, with a non-linear measurement instrument before calibration (i.e., not exactly equal units), does not correlate with QM theory which applies a linear measurement apparatus (equal units) without calibration. To resolve this discrepancy rigorously, empirical metrology measurement results and QM measurement results in theory are both correlated using ordinals and cardinals from Cantor's Transfinite Set Theory (CTST).<sup>14</sup>

### 3.0 Cantor's Transfinite Set Theory (CTST)

CTST is based upon elements with a common, but not defined as equal, property that determines their set membership. "Since every single element  $m$ , if we abstract from its nature, becomes a 'unit', the cardinal number  $\bar{\bar{M}}$  is a definite aggregate<sup>ii</sup> composed of units, and this number has existence in our mind as an intellectual image or projection of the given aggregate  $M$ ."<sup>15</sup> I.e., Cantor defines the highest cardinal of a set as the set's reference.

This paper proposes that Maxwell's unit is a set theoretic ordinal, which is a set member, representing a quantized physical property between two contiguous cardinals. A cardinal does not have a property that determines set membership. Therefore this paper proposes that Maxwell's numerical value is a set theoretic cardinal which is not a member of an ordinal set, but is one limit of the set, and is that set's reference.

### 3.1 Finite sets (QM and metrology practice)

Finite sets are measured in practice using metrology. Fig. 1 correlates a metrology measurement instrument scale to a set of ordinals (e.g.,  $u_1 - u_8$ ). Cantor identified  $m$  as an element of the set  $\bar{\bar{M}}$ .<sup>iii</sup> In this paper  $m$  is replaced with  $u_i$  to represent ordinals with the property ( $u$ ) and their numerical order (sub  $i$ , where each number  $i$  identifies one quantized ordinal in the set). In Fig. 1 the cardinal  $\bar{\bar{8}}$  is the set reference as Cantor proposed.

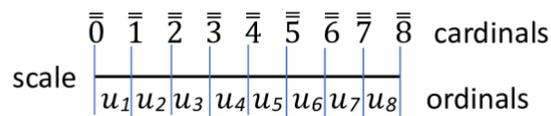


Fig. 1 A measurement apparatus scale and a finite set

A physical measuring stick is an example of Fig. 1. Without any other assumptions the measuring stick is divided by vertical marks into 100 ( $n$ ) independent lengths each,  $u_{1-100}$ . Each vertical mark is a cardinal. Between each set of two contiguous marks, each  $u_{1-100}$ , an ordinal, has a precision determined by calibration relative to a unit standard in a coordinate system.

<sup>ii</sup> Cantor uses "aggregate," the modern usage is "set."

<sup>iii</sup> While  $\bar{\bar{M}}$  is not used further, the two over bars are applied in this paper to identify cardinality .

In this example there is no assumption that  $u_{1-100}$  are each equal to each other before calibration. This example identifies that the calibration of a measurement instrument is required in theory to establish a linear measurement scale.

When the measuring stick in the above example does represent a meter stick with 100 centimeters ( $n$ ), it is common and easier to calibrate to a meter unit standard, rather than to a centimeter unit standard. However, when  $n$  is two, e.g., in QM spin measurement comparisons, individual unit calibration is required which identifies how such QM measurements are different from common metrology measurements.

### 3.2 Transfinite sets (QM theory)

Transfinite sets are measured in QM theory using linear algebra. Cantor recognized the difference between finite sets (metrology or QM practice) and transfinite sets (QM theory): "C. Every finite aggregate E is such that it is equivalent to none of its parts... D. Every transfinite aggregate T is such that it has parts  $T_1$  which are equivalent to it".<sup>16</sup> Cantor is stating that ordinals are finite in physical sets and are infinitesimal in transfinite sets. In transfinite sets the contiguous cardinals T and  $T_1^{\text{iv}}$  are a superposition and the infinitesimal ordinals are equal (linear set). Linear algebra, as applied in the QM wave equation, represents transfinite sets beautifully.

Applying CTST, this paper proposes that a QM eigenvector<sup>v</sup> is equivalent to a cardinal (eigenvalue) and an infinitesimal ordinal. Transfinite measurements, sums of infinitesimal ordinals, become relative to a reference in finite systems.<sup>17</sup> When the infinitesimal ordinals in current QM theory are one, the reference relationship of the infinitesimal ordinal is removed (i.e., one is not relative to a reference) and the infinitesimal ordinals are only locally relative. When the infinitesimal ordinals are not relative to an independent reference only probabilistic sums of eigenvectors (QM measurement results) appear.

### 4.0 Metrology standards in coordinate systems (defines independent references)

A finite set has a limit. Cantor defines the limit of a finite set to be its reference. When references are coordinated by a society they may become standards (independent bounds) such as the SI unit standards.

Based upon CTST, a measurement instrument unit or an exact SI unit standard in a coordinate system can be defined as:

An arbitrary numerical value limit (cardinal) of the units of an identified quantized property (one or more ordinals) relative to or on a coordinate system. All units, either the inexact measurement instrument units or exact unit standards, are ordinals with cardinal limits. On

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<sup>iv</sup> Cardinals may be seen as equivalent to eigenvalues in linear algebra.

<sup>v</sup> A QM eigenvector with a unitized unit is equivalent to Maxwell's quantity.

one linear coordinate axis, the same units (i.e., a unit standard or factor thereof) have equal numerical values by definition which are not arbitrary after their first use.

For example: A Planck, the intrinsic unit of six SI standards, is a numerical value (cardinal) and is also the smallest possible finite ordinal. The ordinal Planck appears to be the boundary between finite and transfinite sets.

Multiple SI unit standards (e.g., meters, kilograms, seconds, etc.) may be identified as one property. Examples: the velocity property is meters/second, or the momentum property is mass (kilograms) times velocity.

### **5.0 Representational measurement theory (without independent references)**

Representational measurement theory (RepMT)<sup>18</sup> is the current widely accepted measurement theory which assumes observations and measurement results are linear (therefore equal units). RepMT provides the measurement theory applied in metrology and QM. In metrology linearity is established when the measurement instrument units (mean set or each independently) are calibrated to the unit standard or factor thereof. However RepMT defines the calibration of a measurement instrument to a coordinate system to be of no consequence in theory.<sup>19</sup> Notice the logical flaw: RepMT requires the measurement instrument linearity established by unit calibration, but excludes unit calibration from the theory.

Erroneously assuming measurement instrument units are equal without calibration, RepMT removes the coordinate system (which supports the calibration of measurement instrument units) from measurement theory. Without an independent coordinate system, measurement results will be locally relative and probabilistic as they appear in QM, not independent and statistical as they appear in metrology after calibration. Without a coordinate system, RepMT also eliminates accuracy and precision relative to the coordinate system.<sup>20</sup> Without the accuracy and precision relative to a coordinate system, QM assumes (erroneously) that metrology measurement results can be exact in theory, even though all high precision repetitive measurement results produce normal distributions in metrology. These omissions in RepMT raise measurement questions.

Following RepMT, metrology erroneously considers an independent unit standard to be empirical. Thus metrology does not recognize that accuracy (of the numerical value) and precision (of the measurement instrument unit) have theoretical impact.

### **6.0 Relative Measurement Theory (with independent references)**

Relative Measurement Theory (RMT)<sup>21</sup> recognizes that independent (non-local) measurement results in theory and practice will always exhibit accuracy and precision when calibrated relative to an independent coordinate system. RMT, as presented in Fig. 2, includes a coordinate system and identifies that calibration, accuracy and precision have theoretical impact. RMT begins the unification of all measurement results.

In RMT a finite one dimensional measurement system (Fig. 2) is defined to include a measurand (in QM an observable<sup>vi</sup>), the measurement apparatus, a unit standard in a coordinate system (as defined in Section 4.0), and to be without any noise (external to the measurement system) or distortion (internal to the measurement system) including distortion of the measurand by an attempted measurement process.

A practical metrology measurement process requires two equal (after calibration) repetitive measurements to be valid. That is, repetitive metrology measurements commute. The RMT definition follows from this metrology practice and produces equal ( $\pm$  accuracy and  $\pm$  precision) repetitive measurement results. When a possible measurand is distorted during an attempted measurement process, as example at the Planck scale, equal repetitive measurement results are not possible. This would be an observation, not a measurement process as defined here.

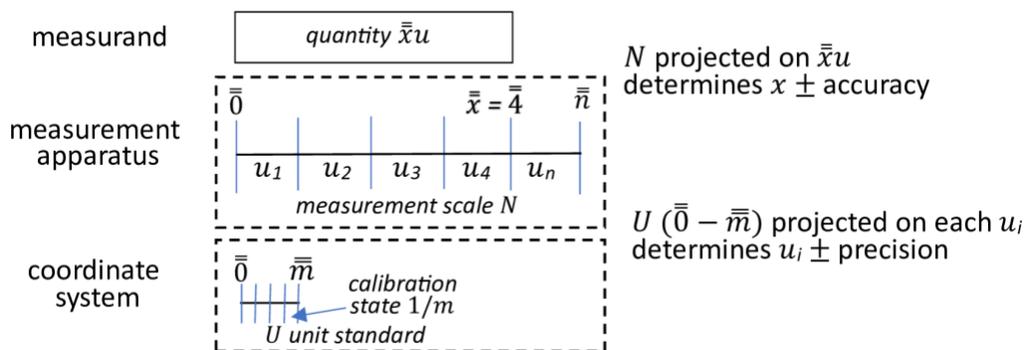


Fig. 2. RMT measurement system

Applying CTST from Fig. 1 to understand Fig. 2: the cardinal  $\bar{4}$  is the numerical value of a measurement result quantity which is the sum of the quantized ordinal elements:  $u_1 + u_2 + u_3 + u_4$ . The projection of  $n$  units (e.g.,  $n = 4$ ) by a measurement apparatus on a measurand causes an entropy increase of  $\log 4$ .<sup>22</sup> This sum of the entropy of each unit of a measurement apparatus verifies that each unit is independent from the others. Neither RepMT, nor metrology, recognize measurement instrument units as independent.

In theory (i.e., without noise or distortion), unit precision is  $\geq$  twice (i.e.,  $\pm$ ) the minimum resolution of the measurement system which is  $\geq h$  (a Planck). In practice, the calibration states ( $1/m$ ) are  $\geq$  the minimum resolution of the measurement system. The calibration states are defined as equal and an SI unit standard ( $U$ ) is defined exactly, then each measurement instrument unit is:

$$u_i = U \pm 1/m \quad (2)$$

<sup>vi</sup> These two terms are different. A measurand can be measured, i.e., it is relative. An observable exists, i.e., it is not relative.

Eq. (2) identifies the theoretical precision ( $\pm 1/m$ ) of each  $u_i$  relative to  $U$ . In this manner the cardinal  $\bar{4}$  is relative to the sum of 4  $u_i$  ordinals each of which is relative to a unit standard  $U$  or a factor thereof. Such a measurement relative to a coordinate system allows for finite independent measurement comparisons.

From Fig. 2 and (2) repetitive one dimensional measurement result quantities produce a distribution:

$$\text{quantities} = \sum_{i=1}^{i=x \pm a} u_i \pm 1/m \quad (3)$$

Equation (3) is proposed as the formal measurement function in a one dimensional coordinate system.<sup>23</sup>

### 6.1 An independent reference allows measurement independence

Following Euler, all measurement result quantities are relative. However there is a difference between locally relative and relative to an independent reference. A physical property ( $u$ ) is relative to independent SI standards when a measurement instrument is calibrated (or assumed to be calibrated) to SI standards. Even the property of a measurand (mass, position, time, etc.) is only rigorously known by calibration to an independent reference. And this independence makes possible independent measurement result comparisons.

All repetitive relative measurement results create distributions. This is verified by the distributions which occur in all QM and metrology repetitive measurements (at high precision).

Consider a brick (a measurand), with multiple unit properties: three space and one time dimension, weight or mass (i.e., particle property), de Broglie wavelength (i.e., wave property), temperature, velocity, etc. The selection of the measurement instrument, calibrated to SI standards (which define each property), determines which property of the brick is measured, including the wave and particle properties.<sup>24</sup>

The QM double slit (identifying a wave) or single hole (identifying a particle) experiments<sup>25</sup> are another example of how the measurement instrument (double slit or single hole) determines the measurand's property. A physical property only appears relative to a measurement instrument which is relative to the reference, which is how a wave or a particle appears on the sensing plate.

This independent relative requirement is formally restated by Tarski's undefinability theorem<sup>26</sup> and Gödel's incompleteness theorems.<sup>27</sup> These theories apply to any formal system (e.g., a measurement apparatus), identifying that comparable measurement results with their relative accuracy and precision (Tarski) or measurement apparatus capable of comparable measurement

results (Gödel) must be relative to non-local coordinate systems. Relative measurements are also verified by all IS-based empirical measurements.

## 6.2 RMT adds independent references to RepMT

Equation (1), quantity =  $xu$ , is appropriate for the counting of linear quantities which are assumed to have equal units (i.e., not relative units) and do not require calibration. Equation (1) is the RepMT measurement function which assumes equal measurement instrument units.

Equation (3) is appropriate for measurements which are relative and independent, therefore non-equal units and require calibration. Measurement results from (3) have a numerical value ( $x$ ) with a theoretical accuracy ( $\pm a$ , see 7.1) and each unit ( $u_i$ ) has a theoretical precision of  $\pm 1/m$  relative to  $U$ . Notice that (1) is equal to (3) when accuracy and precision are removed. Because RepMT actually defines counts, not measurements, the author proposes that RepMT be changed to Relative Measurement Theory.

## 7.0 Measurement mechanics (the application of independent references)

RMT provides the basis to unify all theoretical and practical repetitive single measurements which previously appeared different at the quantum scale. Supporting this unification, Measurement Mechanics<sup>28</sup> previously presented a number of new definitions as the basis for changes to VIM.<sup>29</sup> Independent references make possible rigorous definitions of calibration, accuracy, precision, statistical summing and normal distributions, which are not considered in RepMT and are not as rigorous in VIM. Detailed explanations of these new definitions follow:

### 7.1 Calibration and accuracy

In a transfinite measurement system when a measurement apparatus is projected onto a superposition of cardinals, the ordinals are infinitesimal and each repetitive projection will select which cardinal of a normal distribution represents the measurand's numerical value.

In a finite measurement system, calibration correlates a measurement scale to an independent reference and establishes a finer linear measurement scale to determine accuracy and precision. In any measurement system when a measurement instrument scale is projected onto a measurand, the minimum calibration state is  $1/m$ . Consider that  $1/m$  separates  $u_x$  from  $u_{x+1}$ . If an end of a measurand is within this  $1/m$  calibration state (rare when  $1/n \gg 1/m$ ) the selection of which of the two contiguous ordinals on a measurement scale represents the measurand's numerical value changes randomly with each repetitive measurement.

There is both a maximum and a minimum cardinal (i.e., two limits) to a one dimensional finite measurand. The variation of the numerical value at each end of a measurand is independent of the other end and is added in this worst case. Then a measurand's numerical value has a worst case theoretical accuracy of:  $a = \pm 2(1/n)$ , where  $1/n$  is the mean  $u_i$ . Making two repetitive

measurements (expected to be equal) to reduce the probability of this worst case accuracy is common in metrology practice.

## 7.2 Unit precision

The size of an ordinal unit ( $u$ ) is limited by two cardinals. These two cardinals delineate an independent unit ( $u_i$ ) which has a precision relative to a unit standard or factor thereof. The calibration of each unit in theory correlates each  $u$  of the measurand, via the measurement instrument, to the unit standard  $U$ , correcting each  $u_i$  to the nearest calibration state ( $1/m$ ). Notice that the calibration of each  $u_i$  causes the two states ( $\pm 1/m$ ) of each  $u_i$  to randomly change with each repetitive measurement.

This calibration of each unit is more precise, but is not necessary when  $n$  is large, as the summing of  $n$  randomly changing  $u_i$  very likely cancels the ( $\pm 1/m$ ) differences in the precision of each  $u_i$ .

## 7.3 Statistical summing of unit precision

The measurement system shown in Fig. 2 and formalized in (3) is without any noise or distortion (i.e., theoretical) yet produces a near normal distribution of measurement results, not an equal repeating result. A verification of (3): making repetitive experimental measurements of a fixed measurement system, with minimum noise and distortion, produces a normal distribution of measurement result quantities when the measurement result precision is  $\pm 1/m$  or greater.

Statistical summing of theoretical unit calibration explains how normal measurement result distributions occur. In (3), each finite measurement result quantity consists of a numerical value (e.g.,  $x = 4$  in Fig. 2) of  $u_i$ . A measurement result's numerical value is the limit of the sum of  $u_1 + u_2 + u_3 + u_4$ . In repetitive measurement processes, each independent  $u_i$  has a 50% probability of the random  $\pm 1/m$  precision even when the measurand, the measurement apparatus and the coordinate system are fixed.

## 7.4 Normal distributions

When  $u_1 + u_2 + u_3 + u_4$  each with random  $\pm 1/m$  precision (2 calibration states), are statistically summed over a large enough number of precise repetitive measurements, a total of  $2^4 = 16$  measurement result quantities occur which approximate a normal (Gaussian) distribution. Each of these summations is one metrology measurement result.

The probability of the random precision of a measurement instrument's units changing a measurement result's quantity is rare when  $1/n \gg 1/m$  and  $n$  is large. Common metrology practice:  $m$  is at least 10 times greater than  $n$ . However, when  $n$  is small (in QM spin measurements  $n = 2$ ), the standard deviation of the units (ordinals) diverges from the mean deviation.<sup>30</sup>

## 7.5 Measurement mechanics revises metrology

The ubiquitous occurrence of normal distributions in repetitive measurements, which are not recognized in VIM, indicate that VIM does not fully define empirical measurements. The new definitions elucidated above and other revised definitions in Measurement Mechanics necessitate a revision of VIM.

## 8.0 RMT correlates with QM theory

In MFQM, von Neumann<sup>31</sup> developed, (4) a statistical operator representing a local measurement process, which he termed Process 1. Process 1 generates a local summation of each inner product statistical projection. The inner product function establishes the new operator ( $\mathbf{U}'$ ) relative to the previous operator ( $\mathbf{U}\phi_i$ ). The two operators represent two repetitive measurement functions relative to each other (i.e., local). The  $\mathbf{U}$  (bold) statistical operator in (4) is unrelated to the  $U$  unit standard used in this paper.

$$\mathbf{U}' = \sum_{i=1}^{i=\infty} (\mathbf{U}\phi_i, \phi_i) P_{[\phi_i]} \quad (4)$$

In (4)  $\mathbf{U}'$  is relative to the previous measurement statistical operator  $\mathbf{U}\phi_i$ , not to a non-local coordinate system. To correlate (4) local and transfinite with (3) non-local and finite, the operator  $\mathbf{U}'$  would represent a finite distribution of repetitive measurement result quantities ( $x, x', x'' \dots$ ) and  $\mathbf{U}\phi_i$ , which  $\mathbf{U}'$  is relative to, is removed. Then the local eigenfunction  $\phi_i$  is equivalent to (2) which is correlated to  $U$ .  $\pm 1/m$  in (3) is a near equivalent to the statistical projection operator  $P_{[\phi_i]}$  in (4), but  $P_{[\phi_i]}$  is not relative to  $U$ .<sup>32</sup> These understandings convert (4) into (5):

$$\text{quantities} = \sum_{i=1}^{i=x} u_i \pm 1/m \quad (5)$$

Given the understandings above and without accuracy ( $a$ ), (3) and (5) are equal indicating that QM measurement theory (4) and the metrology measurement theory proposed here (3) are equivalent. The equivalence of the calibration state ( $1/m$ ) of (5) with the QM state  $h$  ( $h$  is the smallest finite physical state) is developed in RMT.<sup>33</sup> As calibration was assumed to be empirical in RepMT, von Neumann did not identify precision ( $\pm 1/m$ ) in (4), but did recognize  $P_{[\phi_i]}$ .

Clearly (5) is not a formal development. Developing non-local QM measurements with the depth von Neumann did for local QM measurements is well beyond the scope of this paper and this author.

The RMT measurement process (3) causes a entropy change of  $H = \log xm$ .<sup>34</sup> This entropy change recognizes the significance of the calibration states ( $1/m$ ) and identifies the entropic effect of a measurement process with calibration.

## 9.0 Measurement mechanics correlates with QM uncertainty

Heisenberg uncertainty appears when an observation process changes the observable. Such QM observations do not commute. In this paper this process is termed a demolition observation, not a metrology measurement.<sup>35</sup> In RepMT, without the concept of observables relative to a coordinate system, the uncertainty of finite observations appears between two conjugate variables.<sup>36</sup> In Heisenberg's uncertainty relation,  $\Delta x \Delta p_x \geq h$ ,<sup>37</sup> the conjugate variables are  $x$  and  $p$  which represent a moving mass with position ( $x$ ) and momentum in the  $x$  direction ( $p_x$ ).

The precision of the position of  $x$  is determined by the frequency of light used to observe it. The higher the frequency of light (unit) the smaller  $\Delta x$  is and the higher the energy of the demolition observation (frequency of light times  $h$  of energy). The greater the energy of this demolition observation the larger is the numerical value of  $\Delta p_x$  (less accurate) relative to the numerical value of the mass's momentum in the first observation ( $mv_{1x}$ ), where  $\Delta p_x = |m(v_{1x} - v_{2x})|$ .

As the above quantity calculus<sup>38</sup> explains, the precision of a unit of position and the accuracy of the numerical value of momentum have an inverse relationship in this demolition observation. When quantity calculus (recognizing units) is applied the inverse relationship of these conjugate variables is explained without Bohr's complementarity.<sup>39</sup>

## 10.0 Resolving the remaining issues

- Occurrence of normal distributions. Statistical summing explains how normal measurement result distributions are caused in theory, not only by empirical noise or distortion. This understanding was not recognized before RMT. RMT answers the question, "Why are Normal Distributions Normal?"<sup>40</sup> The ubiquitous occurrence of normal distributions in metrology measurements (without significant noise or distortion) verifies the statistical summing of  $u_i$ , the precision of each  $u_i$ , theoretical unit calibration, and eq. (3).

- Superluminal transfers (faster than the velocity of light) are also described as: quantum entanglement or quantum teleportation. Quantum entanglement, without superluminal transfers, is used for quantum communications. The apparent superluminal transfers between two entangled neutrons,<sup>41</sup> well described by Mermin,<sup>42</sup> have been shown to be an illusion.<sup>43</sup>

Superluminal transfers, which are not physically possible at any scale, nonetheless gained credibility when multiple experiments verified RepMT. This credibility is not deserved. Both RepMT and these experiments agree because they both erroneously do not recognize the effect of the calibration process between the two measurement instruments which creates the illusion of an information transfer. This analysis is supported by non Planck scale measurements experiments<sup>44</sup> which also do not recognize the calibration process effect and have similar results.

- Compton and Simon cloud chamber experiments.<sup>45, 46</sup> The fixed relationship between two particles after a collision in these cloud chamber experiments is different from the

probabilistic results of repetitive QM measurement practice and theory. The cloud chamber photos monitor the paths of photons and electrons after a collision. This is not two repetitive measurements of the same property. Since the paths of the photon and electron are local (in the cloud chamber) the second path is predetermined by the first.

- Schrödinger's cat.<sup>47</sup> When the unit calibration process is recognized as part of measurement theory, the imprecisely calibrated measurement system proposed by Schrödinger is no longer a paradox.<sup>48</sup>

- Quantum cheshire cat.<sup>49</sup> Since only numerical values, not units, are recognized in QM interferometer experiments, a measurement result's unit (spin) appears as separate from the result's numerical value (up or down). Amusingly, this is similar to the cheshire cat's smile being separate from the cat in Alice in Wonderland.<sup>50</sup>

- Wave function collapse and decoherence.<sup>51</sup> As all repetitive measurements are shown to create distributions there is no reason to consider wave function collapse. Decoherence formally develops that repetitive QM measurement results form a distribution. QM decoherence, the summation of the QM measurement's eigenvectors with error terms, has the same summation form as statistical summing in metrology measurements which also identifies how specific measurement result quantities appear.

- Hidden variables<sup>52</sup> which are not possible locally,<sup>53</sup> are better postulated as a non-local structure. The non-local structure identified by J. S. Bell<sup>54</sup> is an independent reference which makes possible comparisons between independent measurement results.

- Measurement distributions. von Neumann did not consider the transition from an observable in one state to a distribution of repetitive measurement results to be causal.<sup>55</sup> Eq. (3) identifies that the change from a measurand to a distribution is causal when calibration to an independent reference is applied in a measurement process.

- Einstein, Podolsky and Rosen<sup>56</sup> formally identified that the wave equation is not a complete description of physical reality. Measurement results relative to an independent reference provide a complete description of physical reality.

## 11.0 Conclusions

Single QM measurements expressed in linear algebra cannot be compared to metrology measurements without calibration to a reference. RepMT (current measurement theory) erroneously attempts this. Relative Measurement Theory (RMT) revises and renames RepMT. Measurement Mechanics, which unifies measurement theory and metrology results, revises metrology which otherwise does not support this unification. With these changes, Cantor's transfinite set theory, the basis of all mathematics, provides the perspective to compare all measurement processes.

The necessity for relative measurements was identified by Euler. However calibration was not recognized as the difference between local relative and independent relative single measurements. It was erroneously assumed that a single measurement result quantity could, in theory, be a single numerical value and a unity unit. With this assumption, the infinitesimal ordinal unit in transfinite measurements was ignored, which established only local measurements in QM theory.

However, unit calibration modifies each independent unit and summing of the modified units causes repetitive single measurement results to vary, as J. S. Bell<sup>57</sup> verified in theory and calibration or normal distributions verify in practice.

Calibration creates the difference between locally relative (QM) and independently relative (metrology) single measurements. But calibration was assumed to be empirical in RepMT and metrology. The calibration of measurement instruments as defined in Measurement Mechanics makes a metrology measurement process linear and relative. Measurement Mechanics changes metrology from an empirical practice to the theory and practice of all measurements. This new theory and practice of measurements verifies that the QM wave equation does not provide a complete description of physical reality just as Einstein, Poldolsky and Rosen formally developed. But by changing RepMT rather than QM, metrology and QM are now correlated.

### **With Appreciation**

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### **References**

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<sup>2</sup> [https://en.wikipedia.org/wiki/Tarski%27s\\_undefinability\\_theorem](https://en.wikipedia.org/wiki/Tarski%27s_undefinability_theorem)

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- <sup>15</sup> *ibid.* page 86.
- <sup>16</sup> *ibid.* page 108.
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- <sup>21</sup> K. Krechmer, *Relative Measurement Theory*.
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